A Jargon-Busting Guide to Volatility Surfaces and Changes in Implied Volatility

April 23, 2008

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Sales Literature

Introduction

- This document will go through the jargon on skew, term structure, changes in implied volatility, and volatility regimes, that we often use in our Derivative Morning Comments. It assumes a basic understanding of simple option terms (e.g. implied volatility, realised volatility etc.).
- The first section discusses how volatility surfaces are constructed. We begin by introducing skew and term structure before constructing the actual volatility surface.
- The second section looks at implied volatility changes. Typically, vol changes are analysed in terms of strike-by-strike moves or movements along the skew.
- Section three will then go on to discuss the various volatility regimes seen in the markets (sticky-strike, sticky-moneyness, and super-sticky strike).

Outline

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A Jargon-Busting Guide to Volatility Surfaces and Changes in Implied Volatility

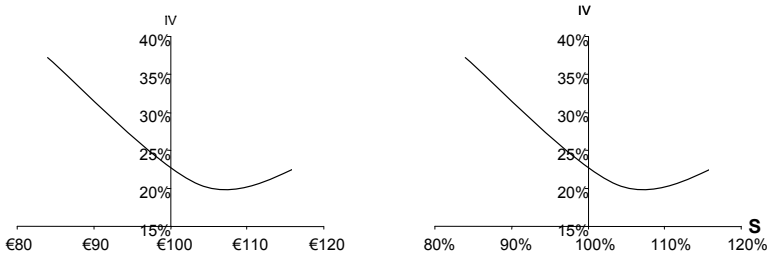
Section I : Constructing a Volatility Surface

- According to the Black-Scholes model, we should see implied volatilities being constant across different strikes and maturities.
- However, in reality the implied volatilities we observe in the markets vary according to strikes and to time, which we call skew and term structure respectively.

a) Skew

What do we mean by skew?

- The skew looks at implied volatilities across different strikes (in currency) or "moneyness" (in % of today's spot level) for a particular expiry.



- The skew is important as it reflects the market's perception of "risk" or "expected volatility" for different potential share price outcomes.
- We tend to see that lower strike puts tend to have higher implied vols in comparison to higher strike puts or ATM options. This is because market participants have been willing to pay relatively more for protection against significant downward movements in stock prices than they have for protection against the same movement to the upside:
    - Empirical evidence - The 1987 crash taught investors that while in the long-run, markets appear to exhibit a negative skew (positive returns), they also appear to have fat tails to the downside (disproportionately high probability of substantial negative returns) – we certainly observe that, while markets tend to grind higher year on year, they also tend to "gap" downwards more often than they "gap" upwards – since options provide protection against such gaps in the market, investors tend to pay more for this protection to the downside.
    - Fear – falling markets can induce an increasing level of uncertainty that leads to higher volatility of stock prices and potentially further declines – particularly in high leverage environments, and where trades are "crowded".
    - Corporate leverage – A declining stock price is usually a reflection of a worsening economic sustainability for a company that at the extreme could lead to bankruptcy. As a result, a lower share price implies greater operating leverage and therefore a greater volatility of the equity component of a company's value (Merton model).
- Hence, in the equity options market, we tend to see implied volatility as a downward sloping curve which bends back upwards. The reason why we see the implied vol lift up for very high strike calls is because these call options represent protection against (or participation in) a *very* significant/violent move upwards. Since this would by necessity tend to be a quite volatile event, this is reflected in the implied vol that is charged for the options.
- This shape of skew is often referred to as a **"smirk".**

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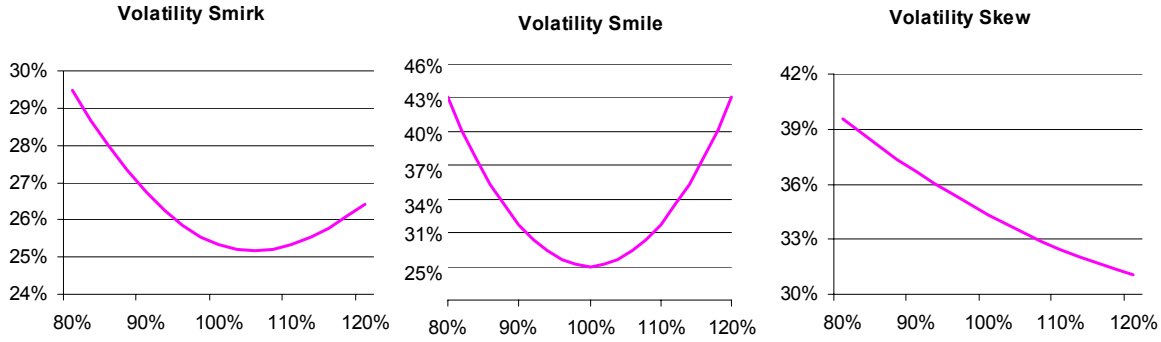
- However, we may see other shapes as well – a perfect volatility **"smile"** is a symmetrical concave-up parabola with the implieds on the wings higher than the ATM vols, and the **"skew"** is referred to a downward sloping curve.



- For the purposes of this document, we will use the term **"skew" in the generic sense** - i.e. varying levels of volatility which across different strikes.

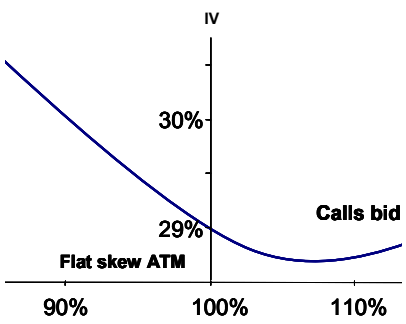
Flat vs. Steep Skew

- Investors using options may often hear that the skew is currently flat or steep.
- Although there is no fixed rule, for a relatively short maturity (e.g. 90-days) we can describe the skew as the difference in implied vols for the 90% and 110% strikes. For a longer maturity (e.g. 730-days), we might choose to look at the 80%-110% strikes (since we can justify looking at strikes that are further out-of-the-money when looking at longer-dated timeframes). What is important here is to pick two strikes between which we think the underlying might have a good chance of moving within the time frame analysed.

Flat Skew

- When we have a flat skew as in the example below, the difference in implied vols of the 90% option and the 110% option is relatively small – here the 90-day skew for KPN shows that the spread is 110bps.
- This suggests that the market does not regard a 10% downward move for KPN as being indicative of a significantly different volatility environment than the one we see today with the shares trading at 100% - i.e. the shares *could* drop 10% (in the same way as they could rise 10%) but if they were to drop 10%, the implied volatility for ATM options (i.e. perception of future riskiness of the shares) would only be expected to rise by a small amount.
- This is why we often see the flattest skews in names with the highest ATM volatilities – since they are already pricing in quite a volatile picture for the shares, and a higher likelihood of moving down to say 90% of current spot in the next 90-days – such a move would not represent a particularly volatile event. In contrast, when ATM implieds are very low, we might expect to see downside put options offered at higher implieds, since a move down to these levels would be a much more significant "event".
- A flat skew (or even **inverted skew** – where call IV is greater than put IV) can also suggest a relatively high market demand for call options vs. that for puts. If an investor were therefore looking to do the opposite (buy puts and sell calls), they would prefer to do so in an environment of flatter skews.

90-day Skew on KPN



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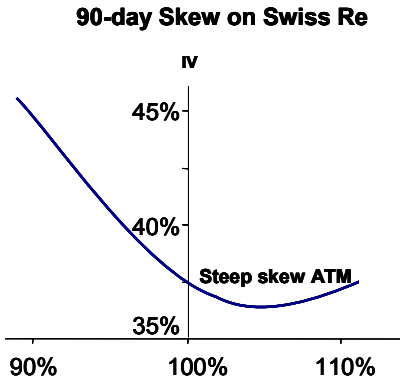
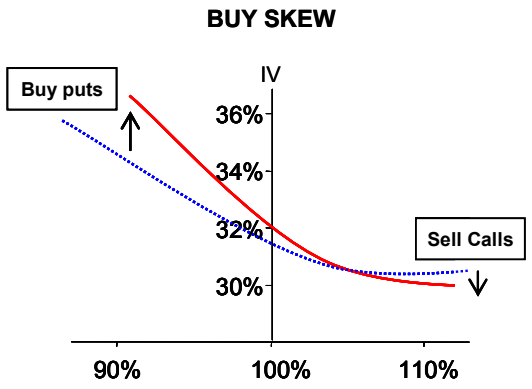
Steep Skew

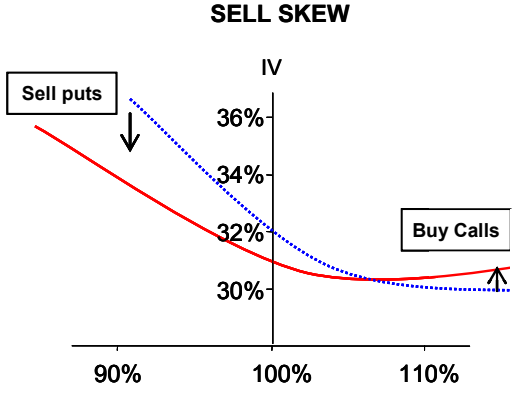
- The same concept is applied for steep skews.
- The graph below shows a simplified 90-day skew for Swiss Re where the 90%-110% spread is c.7% (vs. 1.1% for KPN above), but with ATM vols relatively similar.
- This suggests that there are greater nerves in the market regarding violent downward moves for Swiss Re than for KPN (i.e. investors are prepared to pay more in implied vol terms for these options).



How can we trade skew?

- Investors can trade skew in either "mark-to-market" terms in "buy and hold" terms.
- *Mark-to-market (implied vs implied)*: Investors expecting implied vols to quickly flatten (i.e. the skew to go back to a less steep profile, without necessarily seeing the underlying move particularly), might choose to sell a put and buy a call - as such they would realise a gain in terms of "mark-to-market" since their short option would be worth less, and their long option would be worth more.
- *Buy and hold (implied vs realised):* Alternatively skew trades can be in terms of implied vol vs. realised vol (i.e. holding the options until expiry). When investors see a relatively flat skew which they believe will steepen in the future, they may want to look at **buying the skew.**
- If the investor buys the put today at an implied vol significantly lower than the level of vol that would be realised at maturity if the market were to drop, and if the investor sells the call at an implied vol level significantly higher than the level of vol that would be realised at maturity if the market were to rally, then the trade may be profitable.
- However, we must note that this trade will be **path dependent** even if delta hedged (i.e. we can only judge the success of such a trading programme over a very large number of trades). The best way to partly isolate this path dependency would be through conditional variance swaps. To sell the skew, investors could sell the 110% down var and buy the 90% up var.
- Note that selling the skew would entail the exact opposite positions as shown on the right hand graph.





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- Below is an example of the flattest and steepest skew names that can be found in the MCTopTens file we send out every day as an attachment to the Morning Comments.

Flattest Skew		
Rank	90 day	90%-100%
1	KPN.AS	0.5%
2	ISP.MI	0.5%
3	LHAG.DE	0.6%
4	CONG.DE	1.0%
5	TLIT.MI	1.1%
6	CAGR.PA	1.2%
7	DTEGn.DE	1.2%
8	FTE.PA	1.2%
9	CELR.PA	1.2%
10	SCMN.VX	1.3%
Average		2.1%

Steepest Skew		
Rank	90 day	90%-100%
107	LLOY.L	3.4%
106	ENEI.MI	3.3%
105	BARC.L	3.3%
104	ING.AS	3.2%
103	DGE.L	3.1%
102	PRTP.PA	3.1%
101	BP.L	3.1%
100	AIRF.PA	3.0%
99	REP.MC	3.0%
98	IBE.MC	3.0%
		2.1%

b) Term Structure

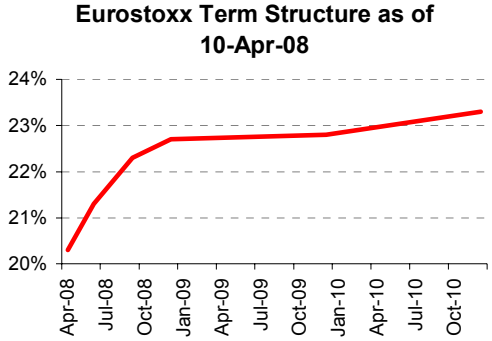
Term structure looks at the implied volatilites for a particular underlying and strike across different maturities. We often describe them as being steep, flat, or inverted.

What do we mean by term structure?

- When using options, it is important not only to consider the volatility skew, but also the term structure. The term structure looks at the implied volatilities for a particular underlying across different maturities.
- In our Morning Comments, we include a table like the one below, which shows the ATM implied volatilities for different maturities.

.STOXX50E		Apr-08	Jun-08	Sep-08	Dec-08	Dec-09	Dec-10
3775.93	9-Apr-08	22.0%	21.4%	22.2%	22.6%	22.8%	23.2%
3758.05	**10-Apr-08**	**20.3%**	**21.3%**	**22.3%**	**22.7%**	**22.8%**	**23.3%**
-0.5%	Total Vol Change	-1.6%	-0.1%	0.1%	0.1%	0.1%	0.2%
Curve last saved	Strike-by-strike	-2.0%	-0.4%	-0.1%	-0.1%	0.0%	0.1%
10-Apr 16:29	Implied by skew	0.4%	0.2%	0.2%	0.1%	0.1%	0.1%

- If we look at the April 10th ATM vols (bold numbers in the row in the middle of the table above), we can see vols increase with maturity so that the "term structure" slopes upwards – see graph below.
- As in the graph, we tend to see that "normal markets" are accompanied by **upward sloping** term structures. This is because of the natural uncertainty associated with longer time horizons – i.e. insurance against flood damage might cost a certain amount for a 1-year policy. The annual fee for a 50-year policy might be expected to be significantly higher though - underwriters have a much better sense of the risk of rising sea levels impacting housing this year than they do 50 years out.



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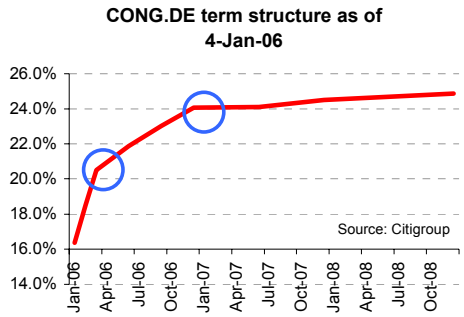
What do we mean by flat, steep, and inverted term structures?

- We talk about flat, steep, and inverted term structures just as we had flat, steep and inverted skews in the previous section.
- For the sake of discussion, we will look at differences in implied volatility between the 12m and the 3m maturity.

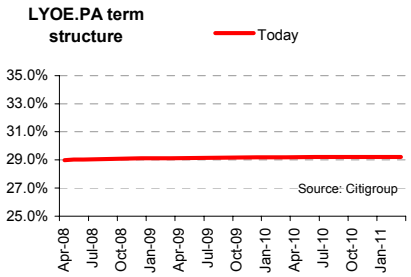
Steep Term Structure

- The graph below plots the term structure of Continental which on the 4th of January 2006 had a 12m-3m implied volatility spread of 3.5%.
- Implied volatilities effectively represent the market's expectation for realised vol out to a certain expiry.
- This relative steepness suggests that the market perceives much less risk/volatility for the short-term, but steadily increasing risk out to the Oct-08 expiry.



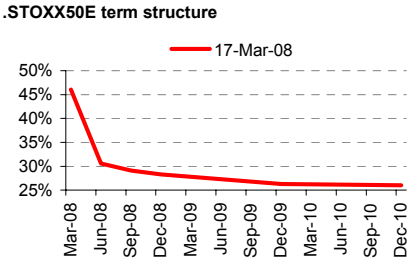
Flat Term Structure

- An example of a flat term structure is given below. The 12m-3m implied vol for LYOE.PA is currently 0%.
- This means that if an investor were to buy an option on LYOE.PA, there would be no extra volatility premium associated with time – i.e. the world/risk-picture is expected to stay relatively consistent for Suez over the next few years.



Inverted Term Structure

- Although in the above cases the term structure was upward sloping, we can also have a **down-ward sloping curve** as well. This is what we call an inverted term structure. As can be seen from the graph below, the term structure on the 17th of March was steeply inverted.
- Such term structure suggests a particularly bad current market environment, such that the implied volatility for the short-dated end is much higher than that for the longer-end. In other words, the market perceives that the current situation looks so bleak that today will be as bad as it will get, and therefore on a time weighted basis, the future looks better, as reflected in the lower vols.



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How do we trade term structure?

- If the current term structure for a particular underlying does not reflect an investor's view, they could engage in a term structure trade.
- Term structure trades are subject to the same subtle difference between MTM trading (implied today vs. implied at a later date), and "buy and hold" trades (implied today vs. realised over the life of the trade).

Term Structure Flattening

- Consider the term structure that we saw for CONG.DE above, which was very steeply upward sloping.
- If an investor saw an imminent risk of M&A that was not priced in to the share price or the options market, then they could buy a short dated call option or variance swap at a relatively low implied vol.
- Often, following an M&A deal we can see share prices stagnate following an initial jump, while the deal gets ratified by shareholders etc (i.e. low vol long term, following an initial high vol period). If an investor sold a long-dated call option or variance swap against the other position, they could lock in a higher implied vol and a higher option premium long dated, while still benefiting in the short-term from their long call option/variance swap.
- We must also mention that there is a path dependency here, just as we saw with the skew trade.

Term Structure Steepening

- Now consider the term structure that we saw for Suez.
- At the moment, the term structure is pricing in the same level of volatility for Suez's share price for 3m and 12m.
- However, suppose that the investor believes that there will be much more realised volatility over the next three months due to uncertainties arising from Suez's merger with Gaz de France, but that the realised vol would come down before the next 12m as we expect clarity on the situation (e.g. expiry of a put-up or shut up). This will be essentially betting that the term structure will go from flat to downward sloping. The investor could then take advantage of the current situation and buy 3m volatility and sell the 12m volatility.

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c) Volatility Surface

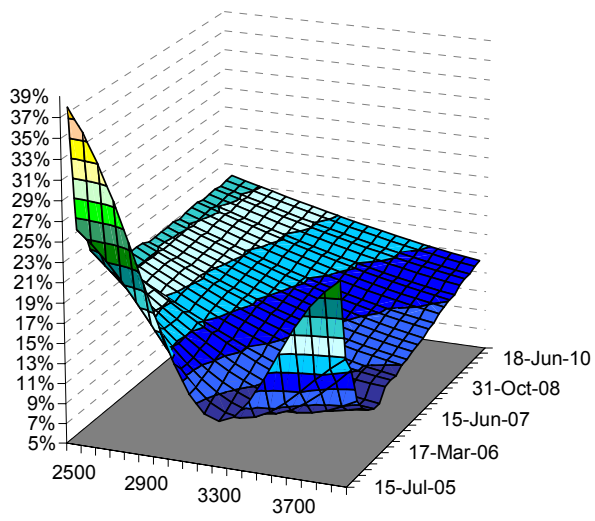
The volatility surface looks at implied volatilities for a given underlying across different strikes and maturities – creating a "snapshot" view that traders can use to assess their risks.

- Now that we have discussed both skew and term structure, we can go on to construct the volatility surface.
- We often see that volatility changes both according to the strike as well as the expiry. If we plot the implied vols as a function of both, then we have the volatility surface.
- Options traders often look at the surface in order to ensure that there are no areas that seem out of place. Furthermore, the surface gives a good snapshot of where the vols are in respect to strikes and expiries.
- Two examples of the surface will be given to show how they can be strikingly different at two different moments in time.

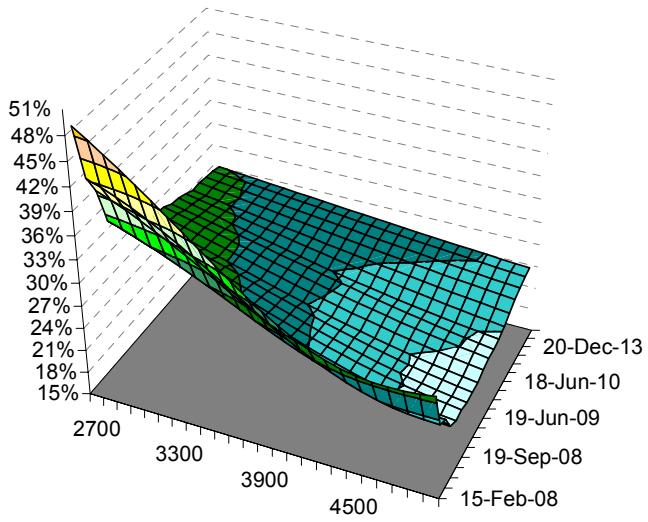
Comparing Two Volatility Surfaces

- The Eurostoxx volatility surface on the 1st of July (bottom left) was typical of what we tend to see during bull markets.
- For simplicity, we will analyse the front-month expiry. The implied vols were highest for deeply OTM puts which then gradually came down around the ATM levels, at which point we saw a bend back upwards in the implieds.
- We have already explained why we tend to see OTM puts price in higher volatility levels.
- We also see that the ATM **term structure was upward sloping**, so that there was more risk priced in for the longer term than the shorter term. This is what we tend to see under "normal" market conditions.

Volatility Surface as of 1-Jul-2005



Volatility Surface as of 22-Jan-08



- The second volatility surface is from the 22nd of January 2008 (above right).
- Here we can see that the skew was still downward sloping for the front month, but less steeply so. This is because ATM implieds already price in a huge amount of volatility – how much worse could the picture get if the market dropped another 10%?
- Furthermore, the **ATM term structure was downward sloping**, reflecting the very poor market conditions that were seen back then.

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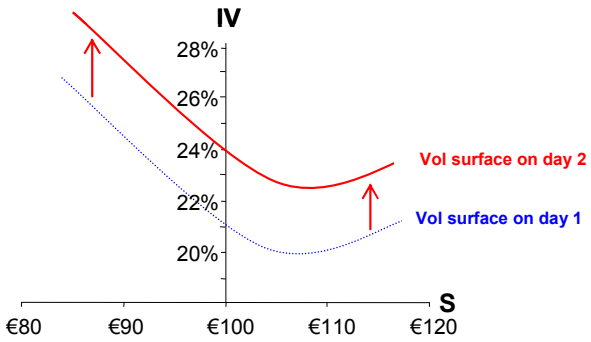
Section II : Changes in Volatility

Changes in volatility can be broken down into two different potential movements which we will explain in turn.

1) Strike-by-strike move – volatility surface shifts
2) Implied by skew – a movement along the skew

What do we mean by strike by strike?

A strike-by-strike move reflects the volatility **surface shifting up/down** i.e. risk perceptions at a given € strike have changed.

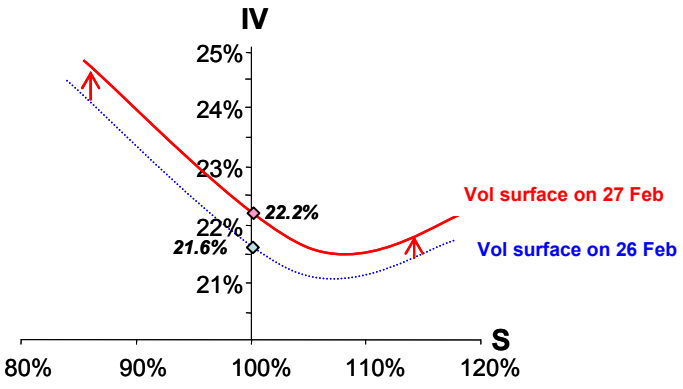


Example

- From the table below, we can see that between the 26th and 27th of Feb, the change in Sep08 ATM implied vols came purely from a strike-by-strike move (by 50bps), since the underlying (SPX) was unchanged on the day.

.SPX		Mar-08	Jun-08	Sep-08	Dec-08	Dec-09	Dec-10
1381.29	26-Feb-08	20.8%	21.8%	21.6%	21.7%	22.0%	22.6%
1380.02	27-Feb-08	21.4%	22.1%	22.2%	22.1%	22.1%	22.6%
-0.1%	Total Vol Change	0.6%	0.3%	0.6%	0.4%	0.1%	0.0%
Curve last saved	Strike-by-strike	0.5%	0.3%	0.5%	0.3%	0.1%	0.0%
27-Feb 19:49	Implied by skew	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%

- The blue curve on the graph below represents the volatility skew on the 26th of February. The ATM implied vol for Sep08 on the SPX index was 21.6%. On the 27th, the index closed relatively unchanged at down 10bps.
- Although there was minimal movement in vol along the skew (since spot didn't really move), we did see a lifting of the surface (i.e. an upward shift movement of the curve).
- The red line therefore shows the new curve where the ATM implied vol (new 100% strike) is now 22.2%.
- This is a "pure" strike-by-strike move.



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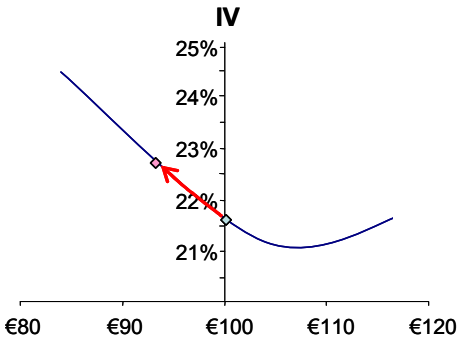
What do we mean by implied by skew?

Implied by skew looks at how much the volatility has moved ***along*** the curve, i.e. how much of the change in ATM implieds that we saw from one day to the next was already predicted by the skew.

- A volatility change that is "implied by skew" reflects a movement along the skew.

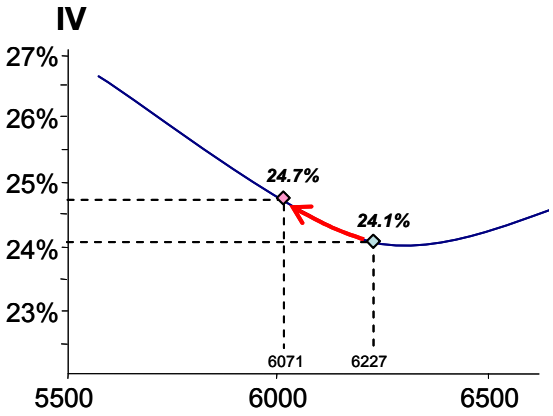


Example

- In the example below, the FTSE index between the 20th and the 21st of November fell by 2.5%.
- Dec-09 ATM implied vol went up from 24.1% to 24.7%, roughly 50 bps.
- None of this change came from strike-by-strike moves but instead from a movement along the skew ("implied by skew") – i.e. the **risk perception of the market at a given strike in € is unchanged.**

.FTSE		Mar-08	Jun-08	Sep-08	Dec-08	Dec-09	Dec-10
6226.50	20-Nov-07	24.7%	24.4%	24.1%	24.3%	24.1%	24.2%
6070.90	21-Nov-07	26.7%	25.7%	25.2%	25.1%	24.7%	24.7%
-2.5%	Total Vol Change	2.0%	1.3%	1.1%	0.8%	0.5%	0.4%
Curve last saved	Strike-by-strike	0.7%	0.3%	0.2%	0.1%	0.0%	0.0%
21-Nov 16:53	Implied by skew	1.3%	1.0%	0.8%	0.7%	0.5%	0.4%

- The blue dot on the graph below indicates where the Dec-09 ATM implied was on the 20th of Nov (24.1%). On the 21st, we saw that the Eurostoxx closed 2.5% lower to 6070.9 (indicated by the pink dot).
- As there was no strike-by-strike move, we know that the 50bps change in the implied vol came just from a movement *along* the skew (with no change in the overall level of the surface itself).
- We indicate this move with the red arrow which shows that the new ATM implied vol (so the new 100% which would be 6070.9) is 24.7%.



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Section III: Volatility Regimes

What is Sticky Strike?

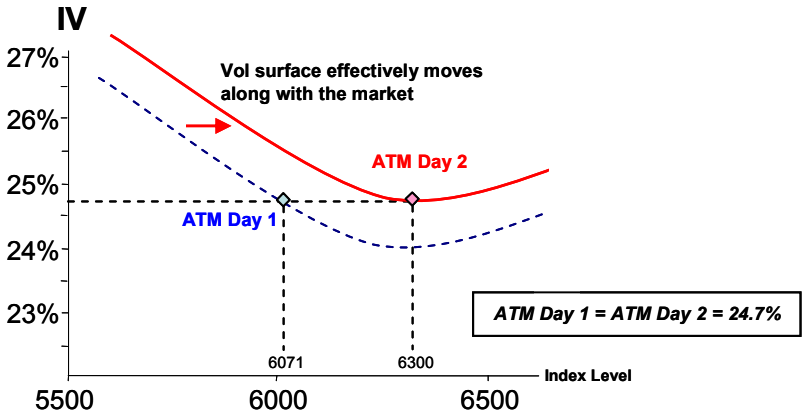
Sticky strike moves occur when we see movements along the skew, but no change in the level of the overall surface. In other words, **for a particular level of the underlying (in say €), the volatility remains constant.**

- Sticky strike refers to the fact that when the volatility changes, this comes purely from a move that is **implied by the skew (as seen above)**.
- Therefore when the spot price changes, the implied volatility given a certain **strike** does not change.
- Using the above graph again, we will continue to associate a 24.1% volatility with the FTSE underlying at 6226.5, and 24.7% for 6070.9 and so on (i.e. the implied is tied to a certain strike in currency/underlying terms).
- We tend to observe sticky strike moves over the short term as the market tends to anchor a certain underlying level with a certain implied vol. We would move away from this sticky strike regime once there has been a genuine rally or sell-off (or change in risk perception).

What is Sticky Moneyness/Delta?

Sticky moneyness occurs when we see a shift of the vol surface. In other words, **for a particular moneyness** (% strike) **or delta** of option (i.e. the option with a 25% delta), the **implied volatility level remains the same.**

- Sticky delta is observed when we see a spot price move where the implied volatility of an option with a given **moneyness/delta** doesn't change.
- From the graph below, we see that on day 1, the underlying is at 6071 and the ATM implied vol is 24.7%.
- On day two, suppose the underlying increases to 6300. This becomes the new 100% or the ATM level.
- However, the volatility for an ATM option remains 24.7%.
- This means that whatever the new underlying price may be, the sticky delta model predicts that the implied volatility for an option will be the same, given a specific moneyness/delta.
- We tend to see sticky moneyness in a longer time horizon, where the perception of implied vols per given moneyness will not change, even if the underlying does (i.e. the FTSE was once trading at c.2000 index points, but with a similar ATM vol to what we see today – the current skew would predict a significantly higher implied vol if the market were to be at 2000 again!).



April 23, 2008

A Jargon-Busting Guide to Volatility Surfaces and Changes in Implied Volatility

What is Super Sticky Strike?

Super-sticky-strike moves are where we **see both movements along the curve as well as a shift in the surface** *in the same direction*. In other words, when the underlying is sold off, the implied volatility goes up by more than the skew would suggest (and the reverse on a rally) - moves in volatility therefore are exaggerated.
Quick hint: On our daily comments, whenever you see **both** the "Strike-by-strike" as well as the "Implied by skew" figures **green** for a particular move (or **both red**), that is a super-sticky strike move.

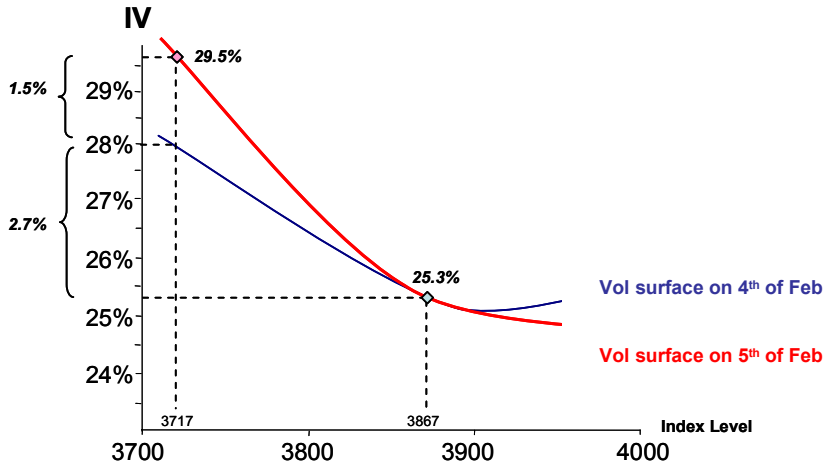
- We observe a super-sticky strike move whenever the vol is bid on a dip in the underlying (or offered on an underlying rally) by more than what is implied by the skew.
- In simpler terms, this means that when the markets go down, we will not only see a movement *along and up* the curve, but also a *shift upward in the vol surface.* Thus we see super sticky strike moves when the markets become very nervous (on a dip) or very optimistic (on a rally) as changes in the implieds are exaggerated.

Example

- The example below shows the Eurostoxx vol levels between the 4th and 5th of Feb, when the index fell by c. 4%.
- Looking at the Mar-08 expiry, we see that there was a 4.2% ATM implied vol change.
- This change was due both from a strike-by-strike (lifting of the surface) move as well as a move implied by the skew (moving along the curve).

.STOXX50E		Feb-08	Mar-08	Jun-08	Dec-09	Dec-10	Dec-11
3867.16	4-Feb-08	27.1%	25.3%	22.7%	22.4%	23.0%	23.6%
3717.08	5-Feb-08	33.9%	29.5%	25.4%	23.5%	23.7%	24.1%
-3.9%	Total Vol Change	6.8%	4.2%	2.7%	1.1%	0.7%	0.5%
Curve last saved	Strike-by-strike	3.2%	1.5%	1.0%	0.4%	0.2%	0.1%
05-Feb 16:16	Implied by skew	3.7%	2.7%	1.7%	0.7%	0.5%	0.4%

- The graph below shows stylised volatility movements (not exact surfaces).
- On 4th Feb the skew is represented by the blue line, the underlying (or ATM level) is at 3867, corresponding to an implied of 25.3%.
- On 5th of Feb, we see the new implied is 29.5%. This came from a 2.7% move up the blue curve (implied by skew) and also a 1.5% lift in the surface (strike by strike).
- If we had seen only a movement along the curve, then the new implied would have been at 28% (25.3%+2.7%).
- However, since we have both movements, the changes in volatility are in a sense **"exaggerated"**, i.e. we see that when the underlying is sold lower, the short dated ATM vol rises by more than the skew would suggest on the dip.
- One can observe a super-sticky strike move when both the "strike-by-strike" and "implied by skew" numbers in the table above are **green or red.**



A Jargon-Busting Guide to Skew and Changes in Volatility

Investors who buy options may lose the premium if the underlying does not move beyond the strike price. If they delta hedge, they may lose money if realised volatility of the underlying is less than implied volatility of the underlying paid in the option. In addition, they are exposed to path-dependent risk.

Investors who buy options may lose their entire premium. Investors who sell options have unlimited risk.

Options involve risk and may not be suitable for all investors. For information on the uses and risks of options, you can obtain a copy of Characteristics and Risks of Standardized Options from Citigroup Global Markets, Options Department, 390 Greenwich Street, New York, NY 10013.